VIVEK TUTORIALS

X (English)

(Chapter Two)

Mathematics Part - II-(2)

DATE: 20-02-19

TIME: 1 Hr

MARKS: 30

SEAT NO:

Q.1 Multiple Choice Questions 2

1 Find the diagonal of a square whose side is 10 cm.

a. $\frac{10}{\sqrt{2}}$ b. $10\sqrt{2}$ c. 100 d. 200

Ans Option b

2 Out of the dates given below which date constitutes a Pythagorean triplet?

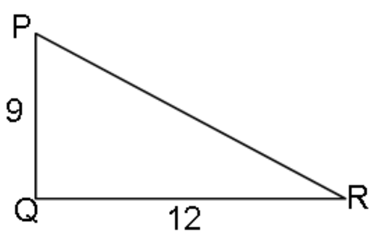
a. 15/08/17 b. 16/08/16 c. 3/5/17 d. 4/9/15

Ans Option a

Q.2 Solve the following 4

1 In the right angled triangle, sides making right angle are 9 cm and 12 cm. Find the length of the hypotenuse

Ans



In $\triangle PQR$, $\angle Q = 90°$

$PR^2 = PQ^2 \times QR^2$ ... (Pythagoras theorem)

$\therefore$ $= 9^2 + 12^2$

$= 81 + 144$

$PR^2 = 225$

$PR = 12$

Hypotenuse = 15 cm

2 In $\triangle LMN$, $l = 5$, $m = 13$, $n = 12$. State whether $\triangle LMN$ is a right angled triangle or not.

Ans $l = 5$, $m = 13$, $n = 12$

$l^2 = 25$, $m = 169$, $n^2 = 144$

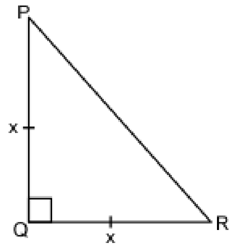
$\therefore$ $169 = 144 + 25$

$\therefore$ $m^2 = l^2 + n^2$

$\therefore$ By Converse of Pythagoras theorem $\triangle LMN$ is a right angled triangle.

Q.3 Attempt the following 4

1 A side of an isosceles right angled triangle is x. Find its hypotenuse.



In $\triangle PQR$, $\angle PQR = 90°$

and $PQ = QR = x$

∴ PR^2 = _____ ... [Pythagoras theorem]

= _____

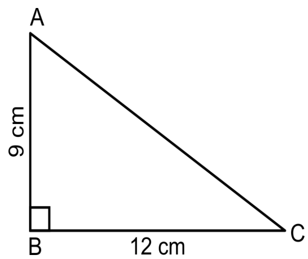
∴ PR^2 = _____

∴ PR = _____units ... [Taking square root]

∴ The length of hypotenuse is _____ units.

Ans 1) $PQ^2 + QR^2$ 2) $x^2 + x^2$ 3) $2x^2$ 4) $\sqrt{2}x$ 5) $\sqrt{2}$ x

2 Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm.



$\triangle ABC$, $\angle ABC = 90°$

AB = 9 cm , BC = 12 cm

By Pythagoras Theorem,

_____ = AC^2

_____ = AC^2

81 + _____ = AC^2

AC^2 = _____

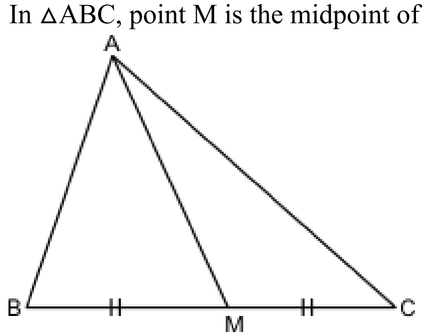
AC = _____

The hypotenuse is _____

Ans 1) $AB^2 + BC^2$ 2) $9^2 + 12^2$ 3) 144 4) 225 5) 15 6) 15 cm.

Q.4 Solve the following 6

1 In $\triangle ABC$, point M is the midpoint of side BC. If, $AB^2 + AC^2 = 290\ cm^2$, AM = 8, find BC



Ans In $\triangle ABC$, seg AM is a median

∴ $AB^2 + AC^2 = 2AM^2 + 2BM^2$ ... [Apollonius theorem]

∴ $290 = 2 \times 8^2 + 2BM^2$

∴ $290 = 2 \times 64 + 2BM^2$

∴ $290 = 128 + 2BM^2$

∴ $2BM^2 = 290 - 128$

∴ $2BM^2 = 162$

∴ $BM^2 = \frac{162}{2}$

∴ $BM^2 = 81$

∴ BM = 9 cm ... [Taking square root]

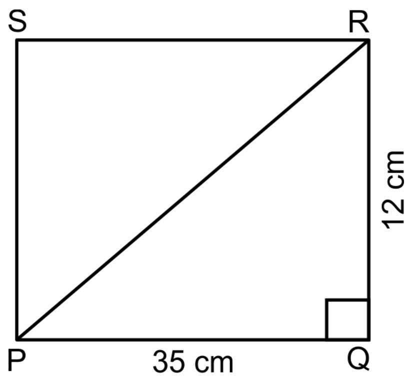
Now BC = 2 BM ... [M is midpoint of side BC]

$= 2 \times 9$

∴ BC = 18 cm

2 Find the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

Ans



In □ PQRS is a rectangle

∴ $\angle Q = 90°$ [Angle of a rectangle]

In $\triangle PQR$, $\angle Q = 90°$

∴ $PR^2 = PQ^2 + QR^2$ ... [Pythagoras theorem]

∴ $PR^2 = 35^2 + 12^2$

∴ $PR^2 = 1225 + 144$

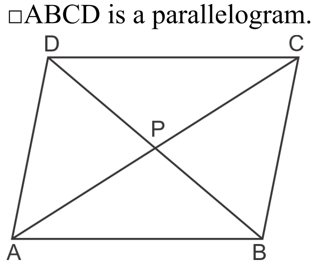
∴ $PR^2 = 1369$

∴ PR = 37cm ... [Taking square root]

∴ Length of diagonal of the rectangle is 37 cm

Q.5 Answer the following 8

1 □ABCD is a parallelogram. Side AB = diagonal BD. Prove that $BD^2 + 2BC^2 = AC^2$.



Ans Proof: The diagonals of a parallelogram bisect each other.

∴ $AP = PC = \frac{1}{2}AC$ and $BP = PD = \frac{1}{2}BD$ ... (1)

In $\triangle ABC$, BP is the median.

∴ by Apollonius' theorem,

$AB^2 + BC^2 = 2BP^2 + 2AP^2$ ... (2)

$AP^2 = \left(\frac{1}{2}AC\right)^2$ ... [From (1)]

∴ $AP^2 = \frac{AC^2}{4}$ ∴ $2AP^2 = \frac{AC^2}{2}$ ... (3)

$BP^2 = \left(\frac{1}{2}BD\right)^2$ ... [From (1)]

∴ ... (4)

$BP^2 = \frac{BD^2}{4}$ $\therefore$ $2BP^2 = \frac{BD^2}{2}$

From (2), (3) and (4),

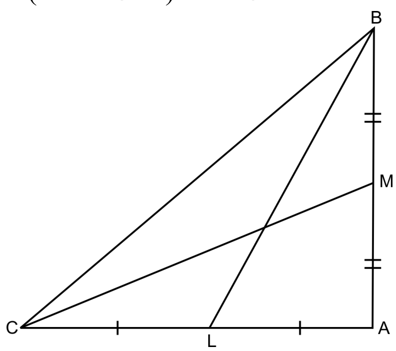
$AB^2 + BC^2 = \frac{AC^2}{2} + \frac{BD^2}{2}$

$\therefore$ $2AB^2 + 2BC^2 = AC^2 + BD^2$ ... (Multiplying both the sides by 2)

$\therefore$ $2BD^2 + 2BC^2 = AC^2 + BD^2$ ... (Given : AB = BD)

$\therefore$ $BD^2 + 2BC^2 = AC^2$

2 In $\triangle ABC$, $\angle BAC = 90°$, seg BL and seg CM are medians of $\triangle ABC$. Then prove that: $4 (BL^2 + CM^2) = 5 BC^2$



Ans Proof:

i. In $\triangle ABC$, $\angle BAC = 90°$ ... [given]

$\therefore$ $BC^2 = AB^2 + AC^2$ ... [Pythagoras theorem]

ii. In $\triangle BAL$, $\angle BAL = 90°$ ... [given, C-L-A]

$\therefore$ $BL^2 = AB^2 + AL^2$ ... [Pythagoras theorem]

$= AB^2 + \left(\frac{1}{2}AC\right)^2$ ... [L is the midpoint of seg

$\therefore$ $BL^2 = AB^2 + \frac{1}{4}AC^2$

$\therefore$ $4 BL^2 = 4 AB^2 + AC^2$ ... [Multiplying both sides by 4]

iii. In $\triangle MAC$, $\angle MAC = 90°$ ... [given, B-M-A]

$\therefore$ $CM^2 = AM^2 + AC^2$ ... [Pythagoras theorem]

$= \left(\frac{1}{2}AB\right)^2 + AC^2$ ... [M is the midpoint of seg A]

$\therefore$ $CM^2 = \frac{1}{4}AM^2 + AC^2$

$\therefore$ $4 CM^2 = AB^2 + 4AC^2$ ... [Multiplying both sides by 4]

iv. $4 BL^2 + 4 CM^2 = 4 AB^2 + AC^2 + AB^2 + 4AC^2$ ... [Adding (2), (3)

$= 5 AB^2 + 5AC^2$

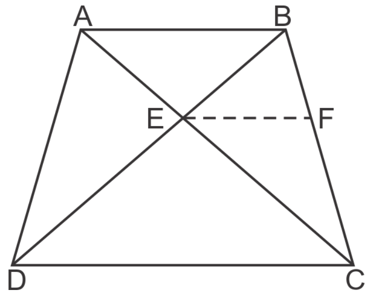
$= 5 (AB^2 + AC^2)$

$\therefore$ $4 (BL^2 + CM^2) = 5 BC^2$ ... [From (1)]

Q.6 Answer the following 6

1 In □ABCD, diagonals AC and BD intersect at the point E. If $\frac{AE}{EC} = \frac{BE}{ED}$, then prove that □ABCD is a trapezium, using basic proportionality theorem.

Ans



Construction:

Draw seg EF || side DC, B-F-C.

Proof: In $\triangle$BDC, seg EF || side DC. ... (Construction)

∴ by basic proportionality theorem,

$\frac{BE}{ED} = \frac{BF}{FC}$ ... (1)

$\frac{BE}{ED} = \frac{AE}{EC}$ ... (Given) ... (2)

From (1) and (2), $\frac{BF}{FC} = \frac{AE}{EC}$

∴ $\frac{FC}{BF} = \frac{EC}{AE}$ ... (By invertendo) ... (3)

In $\triangle$CAB, $\frac{FC}{BF} = \frac{EC}{AE}$ ... [From (3)]

∴ by converse of BPT,

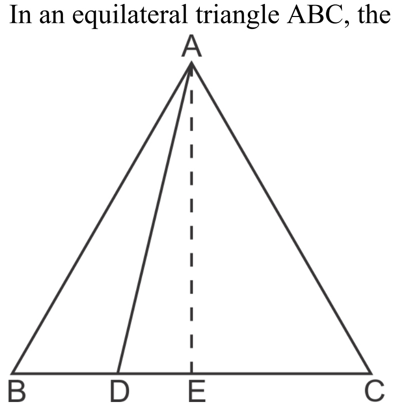
EF || AB

EF || AB ... (Proved) and EF || DC... (Construction)

∴ AB || DC ... (Lines parallel to the same line)

∴ □ABCD is a trapezium.

2 In an equilateral triangle ABC, the side BC is trisected at D. Prove that $9\ AD^2 = 7AB^2$.



Ans Construction : Draw seg AE ⊥ side BC.

Proof : In $\triangle$ABE, $\angle AEB = 90°$ ... (Construction)

$\angle ABE = 60°$ (Angle of an equilateral triangle)

∴ the remaining $\angle BAE = 30°$.

$\triangle$ABE is a 30° - 60° - 90° triangle.

∴ $AE = \frac{\sqrt{3}}{2} AB$ ... (Side opposite to 60°) ... (1)

and $BE = \frac{1}{2} AB$ ... (Side opposite to 30°)

∴ $BE = \frac{1}{2} BC$ ... (Side of an equilateral triangle) ... (2)

DE = BE - BD ... (B-D-E)

$= \frac{1}{2} BC - \frac{1}{3} BC$ ... [From (2) and D is the point of trisection of side BC]

$= \frac{3BC - 2BC}{6} = \frac{BC}{6}$

$\therefore$ $DE = \frac{AB}{6}$ ... (BC = AB = AC) ... (3)

In right angled $\triangle ADE$, by Pythagoras' theorem,

$AD^2 = AE^2 + DE^2$

$= \left(\frac{\sqrt{3}}{2}AB\right)^2 + \left(\frac{AB}{6}\right)^2$ ... [From (1) and (3)]

$= \frac{3}{4} AB^2 + \frac{1}{36}AB^2$

$\therefore$ $AD^2 = \frac{27AB^2 + AB^2}{36}$

$= \frac{28AB^2}{36}$

$\therefore$ $AD^2 = \frac{7AB^2}{9}$

$\therefore$ $9AD^2 = 7\,AB^2$